Probability and Random Processes

ECS 315

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11 Multiple Random Variables

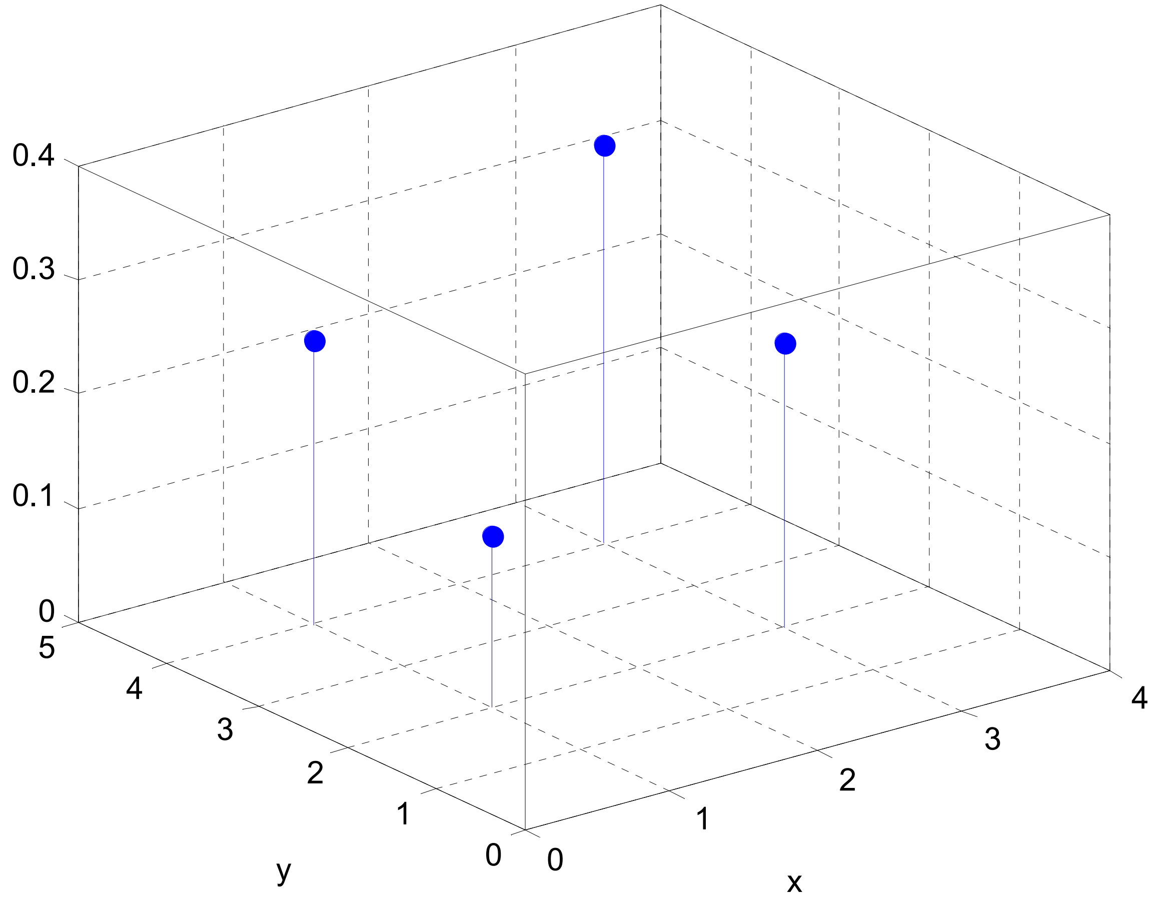
Office Hours:

BKD, 6th floor of Sirindhralai building

Tuesday	**9:00-10:00**
Wednesday	**14:20-15:20**
Thursday	**9:00-10:00**

Example: small joint pmf matrix Ex. 11.9

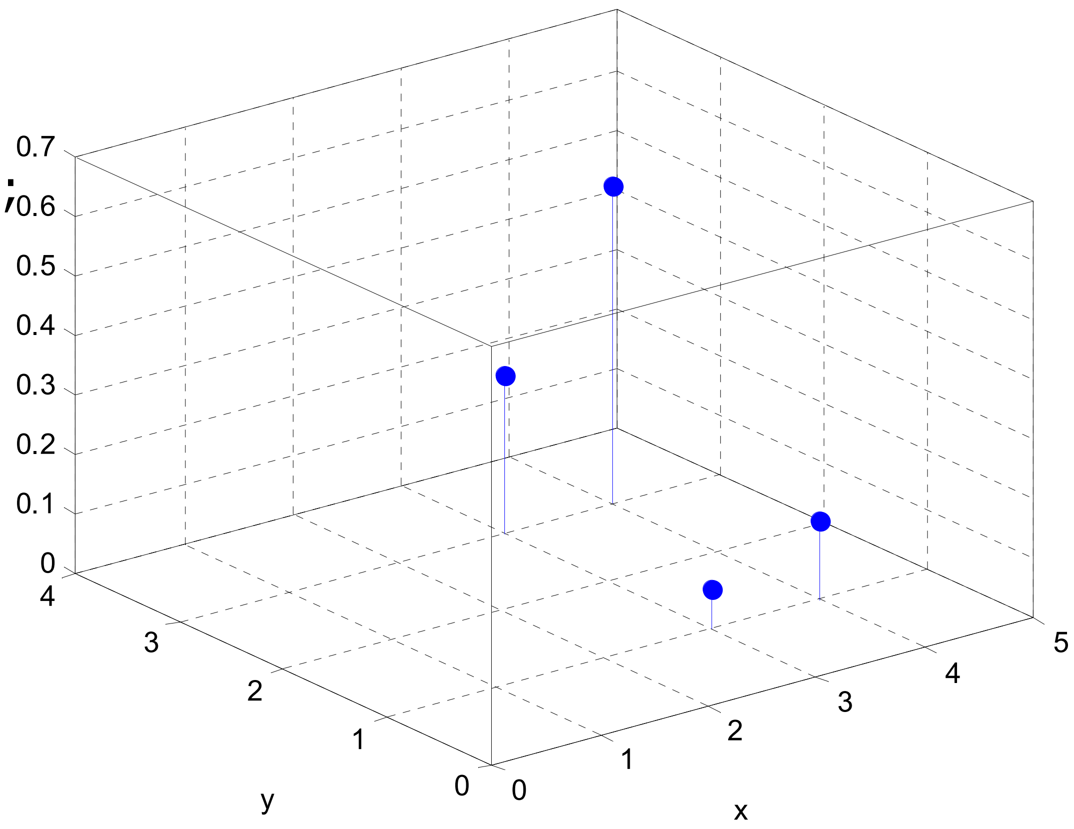
```
close all; clear all;
x = [1 3];
y = [2 4];
PXY = [3/20 5/20; 5/20 7/20];

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY,'filled')
xlim([0,4])
ylim([0,5])
xlabel('x')
ylabel('y')
```



(More)

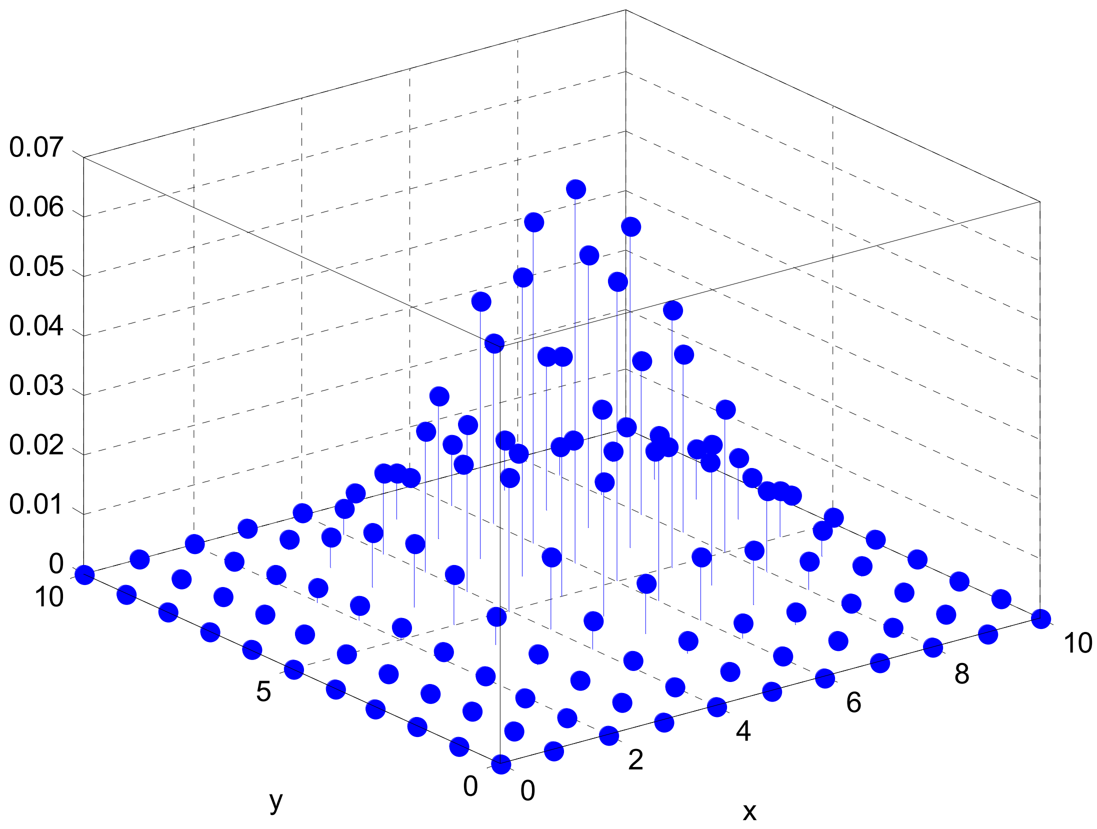
Example: small joint pmf matrix Ex. 11.29

```
close all; clear all;
x = [3 4];
y = [1 3];
PXY = [1/15 4/15; 2/15 8/15];

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY,'filled')
xlim([0,5])
ylim([0,4])
xlabel('x')
ylabel('y')
```



Example: large joint pmf matrix

```
close all; clear all;
n = 10; p = 3/5;
x = 0:n;
y = 0:n;

pX = binopdf(x,n,p);
pY = binopdf(y,n,p);

PXY = pX.'*pY;

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY, 'filled')
%mesh(X,Y,PXY)
%surf(X,Y,PXY)

xlabel('x')
ylabel('y')
```



Evaluation of Probability

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X + Y < 7]$

Evaluation of Probability

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X+Y<7]$

Step 1: Find the pairs (x,y) that satisfy the condition "$x+y < 7$"

One way to do this is to first construct the matrix of $x+y$.

$x+y$

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

Evaluation of Probability

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X + Y < 7]$

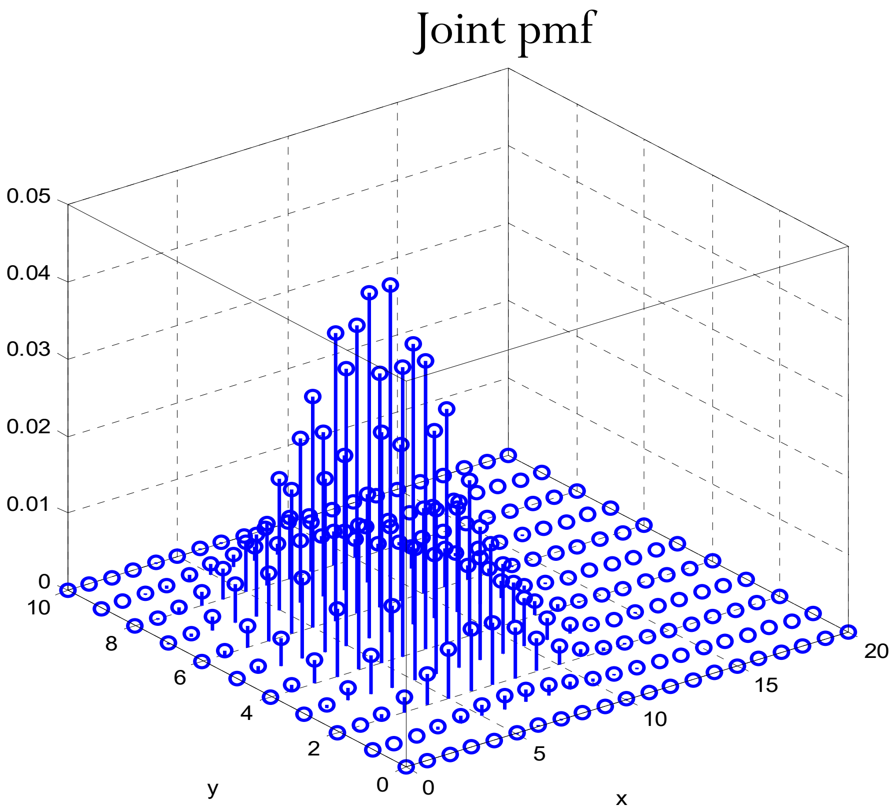
Step 2: Add the corresponding probabilities from the joint pmf (matrix)

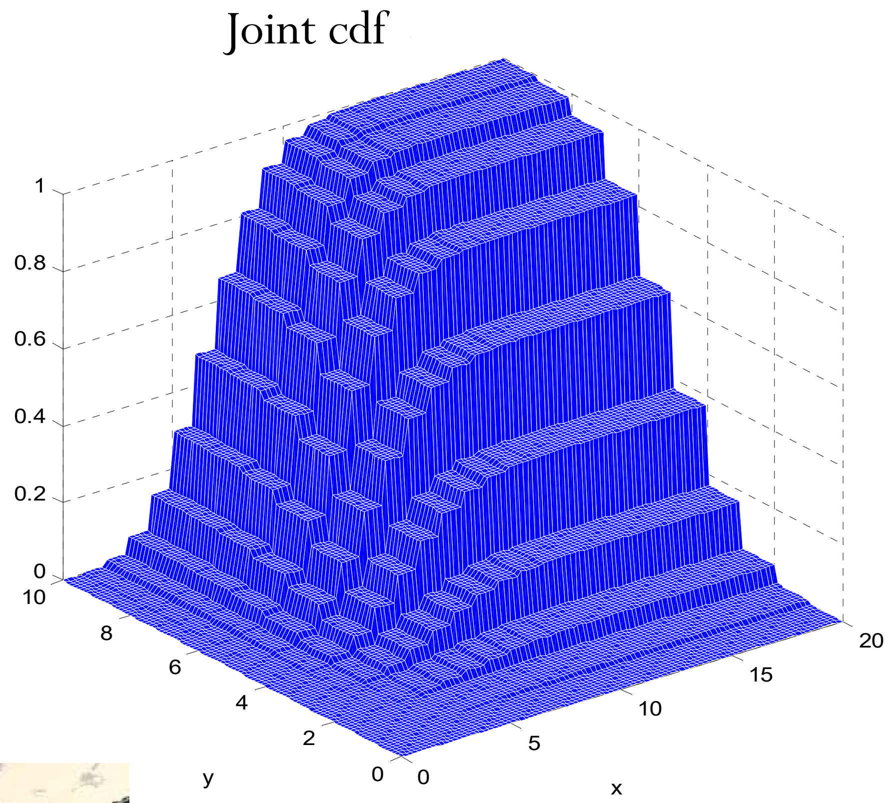
$x + y$

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

$$P[X + Y < 7] = 0.1 + 0.1 + 0.1 = 0.3$$

Example: Joint pmf and joint cdf



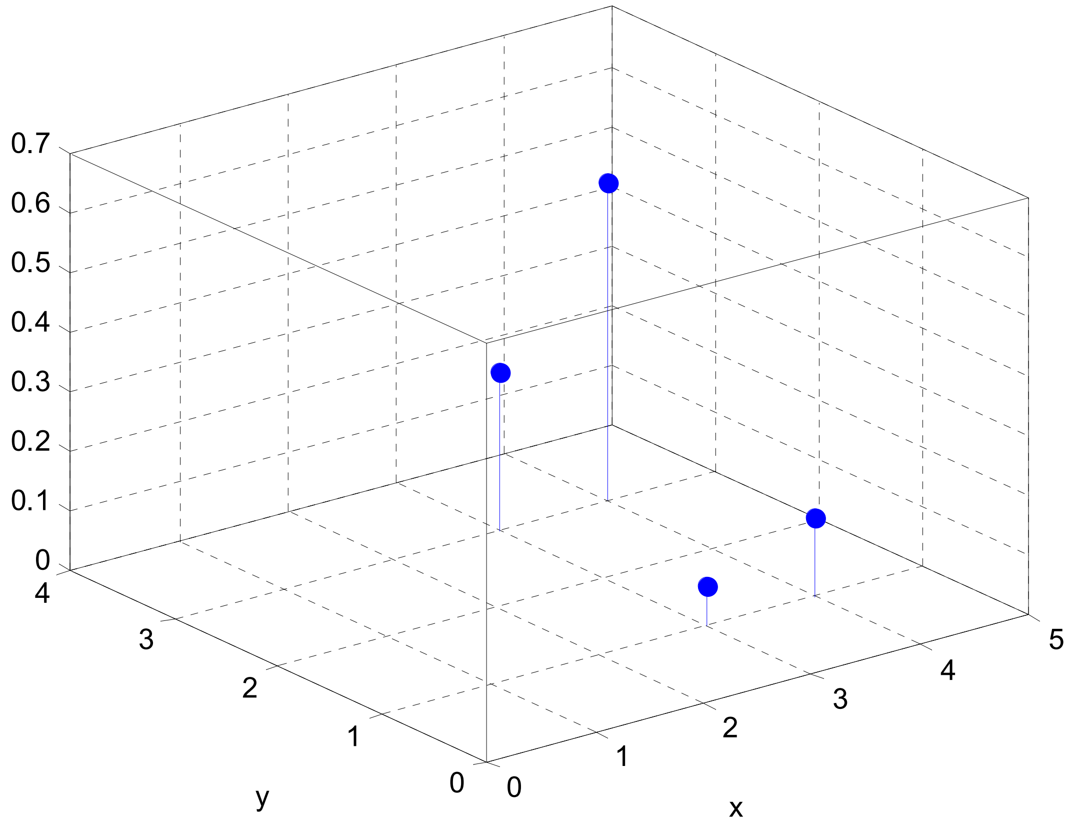


Example: small joint pmf matrix

Ex. 11.29

$$P_{X,Y} = \begin{array}{c|cc} x \backslash y & 1 & 3 \\ 3 & 1/15 & 4/15 \\ 4 & 2/15 & 8/15 \end{array}$$

```
close all; clear all;
x = [3 4];
y = [1 3];
PXY = [1/15 4/15; 2/15 8/15];

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY,'filled')
xlim([0,5])
ylim([0,4])
xlabel('x')
ylabel('y')
```



Joint pmf matrix for independent RVs

Command Window

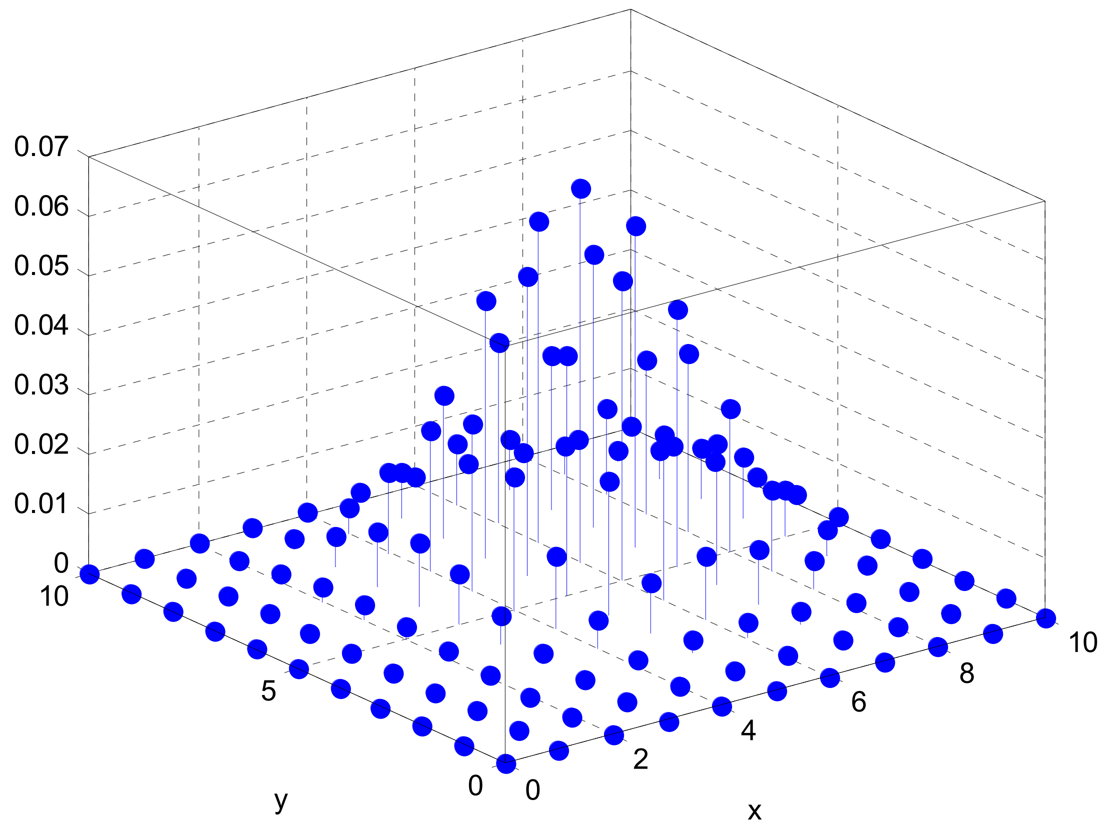
```
>> pX = [1/3 2/3]
pX =
    0.3333    0.6667
>> pY = [1/5 4/5]
pY =
    0.2000    0.8000
>> sym(pX'*pY)
ans =
[ 1/15, 4/15]
[ 2/15, 8/15]
>>
```

Joint pmf for two i.i.d. RVs

```
close all; clear all;
n = 10; p = 3/5;
x = 0:n;
y = 0:n;

pX = binopdf(x,n,p);
pY = binopdf(y,n,p);

PXY = pX.'*pY;

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

%stem3(X,Y,PXY, 'filled')
mesh(X,Y,PXY)
%surf(X,Y,PXY)

xlabel('x')
ylabel('y')
```

Note how the pmfs are multiplied because of the independence.

$$X, Y \overset{\text{i.i.d.}}{\sim} \mathcal{B}\left(10, \frac{3}{5}\right)$$



Sum of two discrete RVs

- Formula-wise:

$$P\left[X+Y<7\right]=\sum_{\substack{(x,y)\\ x+y<7}} p_{X,Y}(x,y)$$

Step 2, Step 1

$x+y$:

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

For our example, only

$$(x,y)\in\left\{\begin{matrix}(1,2),(1,3),(1,4),\\(1,5),(3,2),(3,3),\\(4,2)\end{matrix}\right\}$$

satisfy the condition

- Alternative way to write this:

$$P\left[X+Y<7\right]=\sum_{x}\sum_{\substack{y\\ x+y<7}} p_{X,Y}(x,y)=\sum_{y}\sum_{\substack{x\\ x+y<7}} p_{X,Y}(x,y)$$

Sum of two discrete RVs

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X + Y = 7]$

Step 1: Find the pairs (x,y) that satisfy the condition "$x+y = 7$"

One way to do this is to first construct the matrix of $x+y$.

$x + y$

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

Sum of two discrete RVs

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X+Y=7]$

Step 2: Add the corresponding probabilities from the joint pmf (matrix)

$x+y$

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

$P[X+Y=7] = 0.1$

Sum of two discrete RVs

- Formula-wise:

$$P[X+Y=7] = \sum_{\substack{(x,y)\\ x+y=7}} p_{X,Y}(x,y)$$

Step 2

Step 1

For our example, only $(x,y) \in \{(1,6),(3,4),(4,3)\}$ satisfy the condition

$x+y$

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

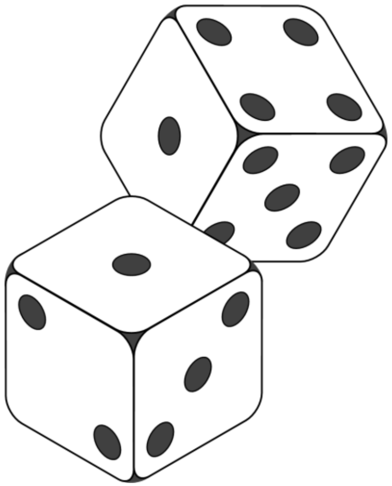
- Other ways to write (and think about) this:

$$P[X+Y=7] = \sum_{x} \sum_{\substack{y\\ x+y=7}} p_{X,Y}(x,y) = \sum_{x} p_{X,Y}(x,7-x)$$

$$= \sum_{y} \sum_{\substack{x\\ x+y=7}} p_{X,Y}(x,y) = \sum_{y} p_{X,Y}(7-y,y)$$

Sum of Two dice

- Assume that the two dice are fair and independent.



DICE CHART		
ROLL		PROBABILITY
2		1/36
3		2/36
4		3/36
5		4/36
6		5/36
7		6/36
8		5/36
9		4/36
10		3/36
11		2/36
12		1/36

Sum of two indp random variables

- = convolution of their pmf

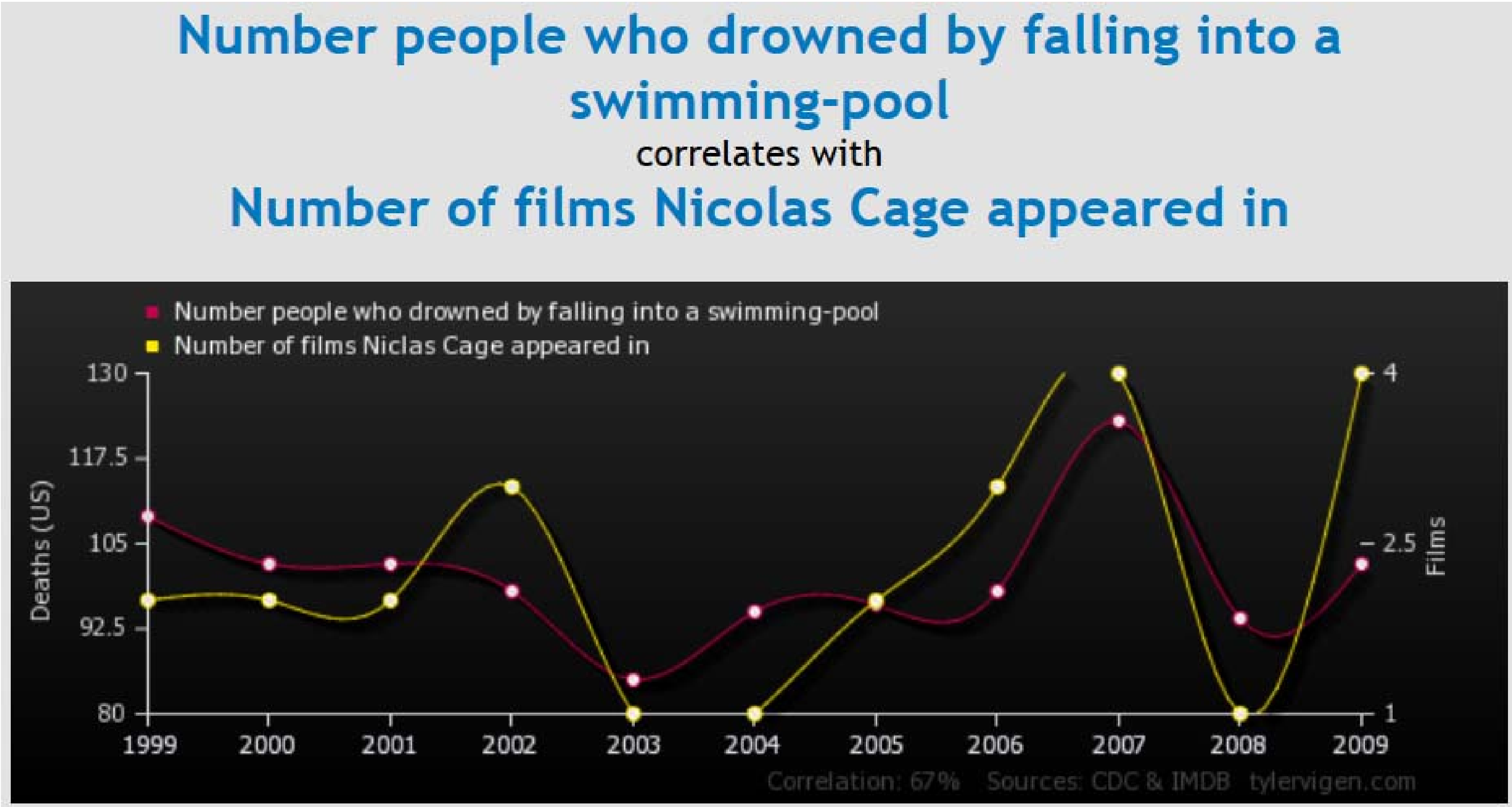
```
>> pX = ones(1,6)/6
pX =
    0.1667    0.1667    0.1667    0.1667    0.1667    0.1667
>> pY = pX
pY =
    0.1667    0.1667    0.1667    0.1667    0.1667    0.1667
>> pZ = conv(pX,pY)
pZ =
  Columns 1 through 7
    0.0278    0.0556    0.0833    0.1111    0.1389    0.1667
0.1389
  Columns 8 through 11
    0.1111    0.0833    0.0556    0.0278
>> sym(pZ)
ans =
[ 1/36, 1/18, 1/12, 1/9, 5/36, 1/6, 5/36, 1/9, 1/12, 1/18, 1/36]
```

Unfortunately, the `conv` command in MATLAB does not work with symbolic numbers.

Correlation

- Correlation measures a specific kind of dependency.
  - Dependence = statistical relationship between two random variables (or two sets of data).
  - Correlation measures "linear" relationship between two random variables.
- Correlation and causality.
  - "Correlation does not imply causation"
  - Correlation cannot be used to infer a causal relationship between the variables.

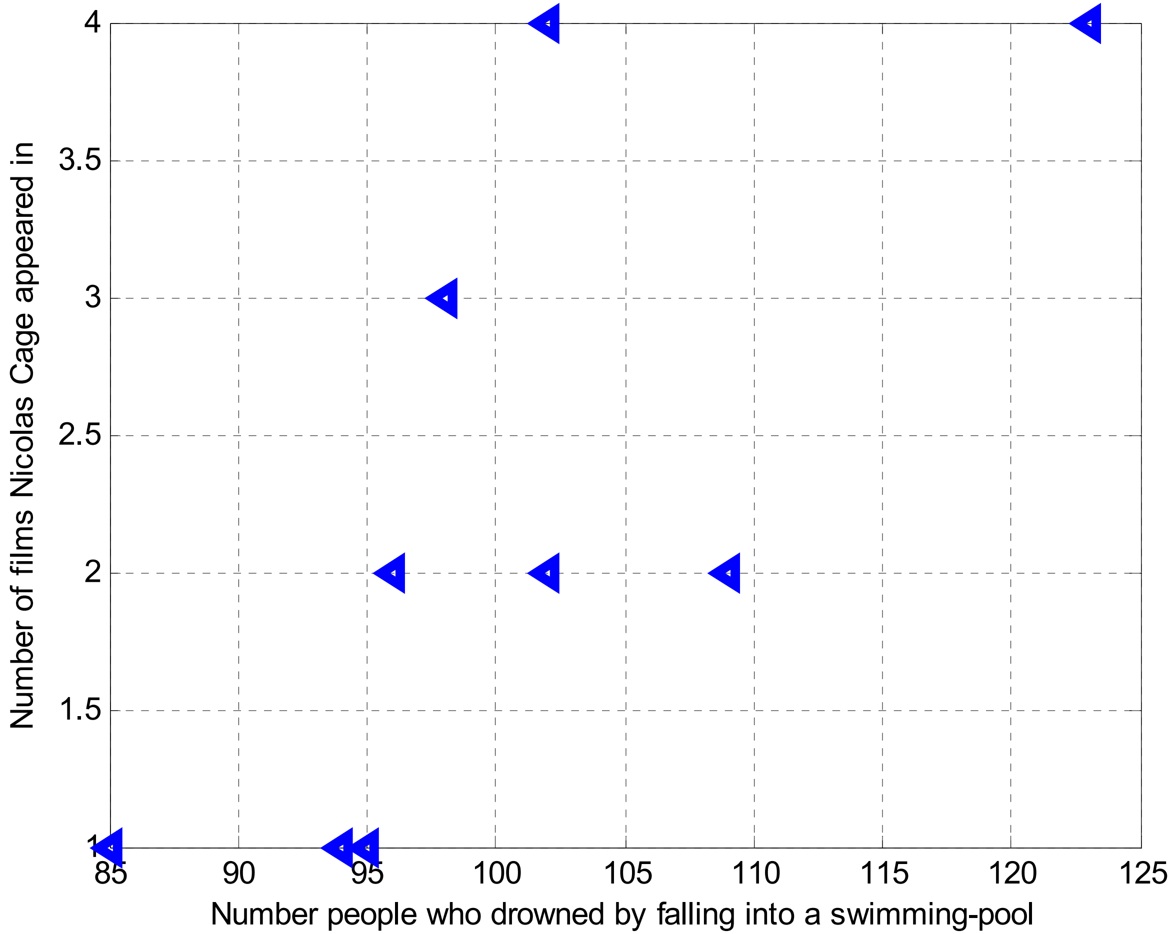
Two "Unrelated" Events



Correlation: 0.666004

http://www.tylervigen.com/

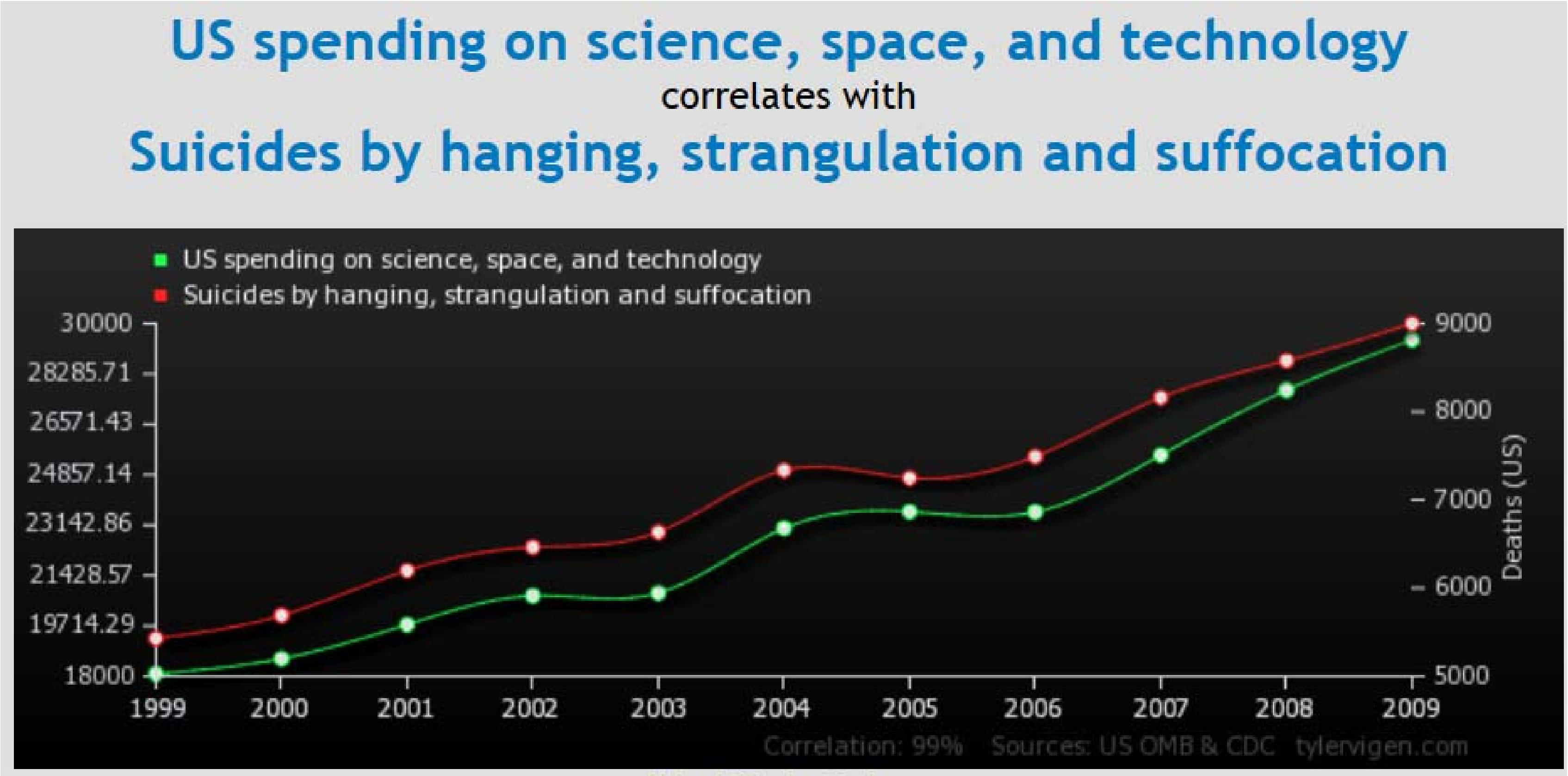
Two “Unrelated” Events



Correlation: 0.666004

http://www.tylervigen.com/

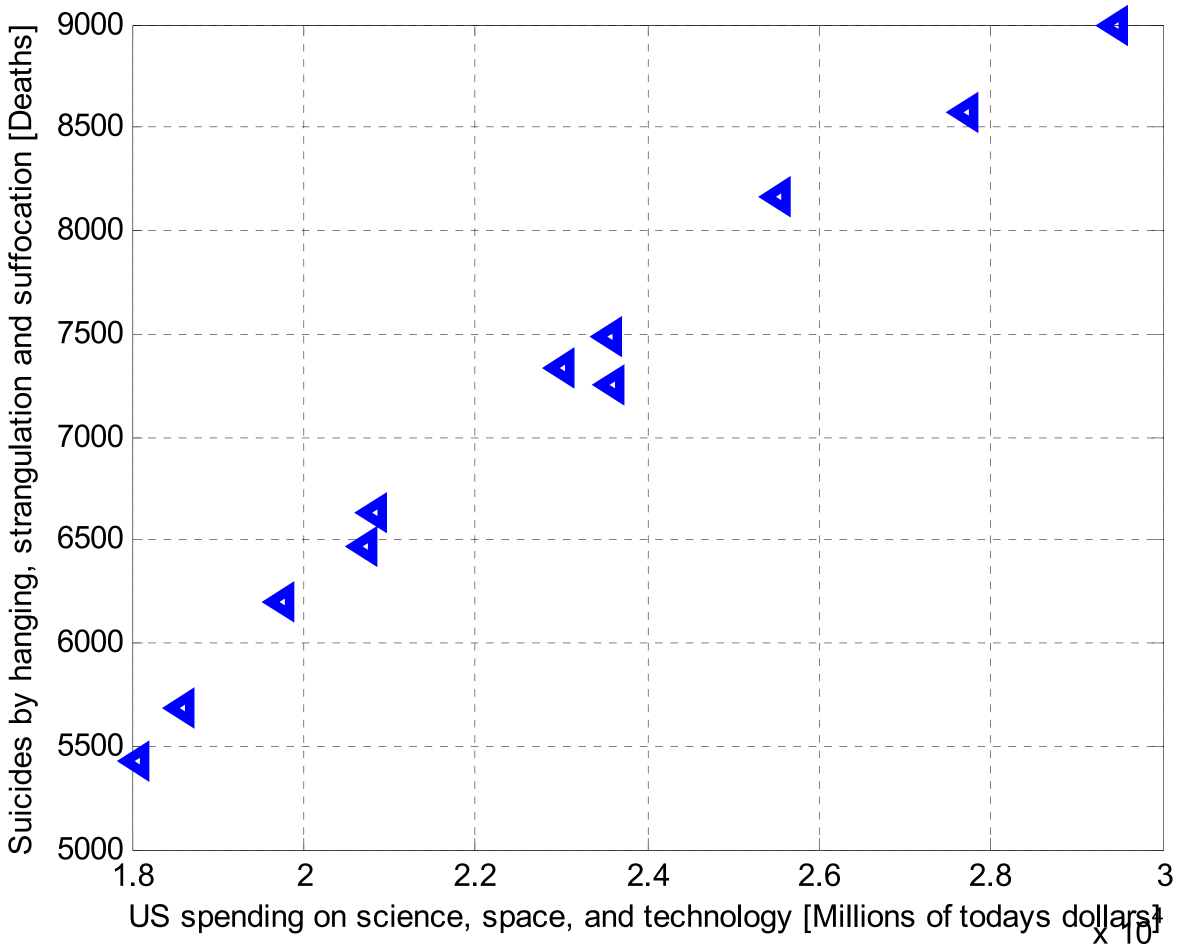
Spurious Correlation



Correlation: 0.992082

http://www.tylervigen.com/

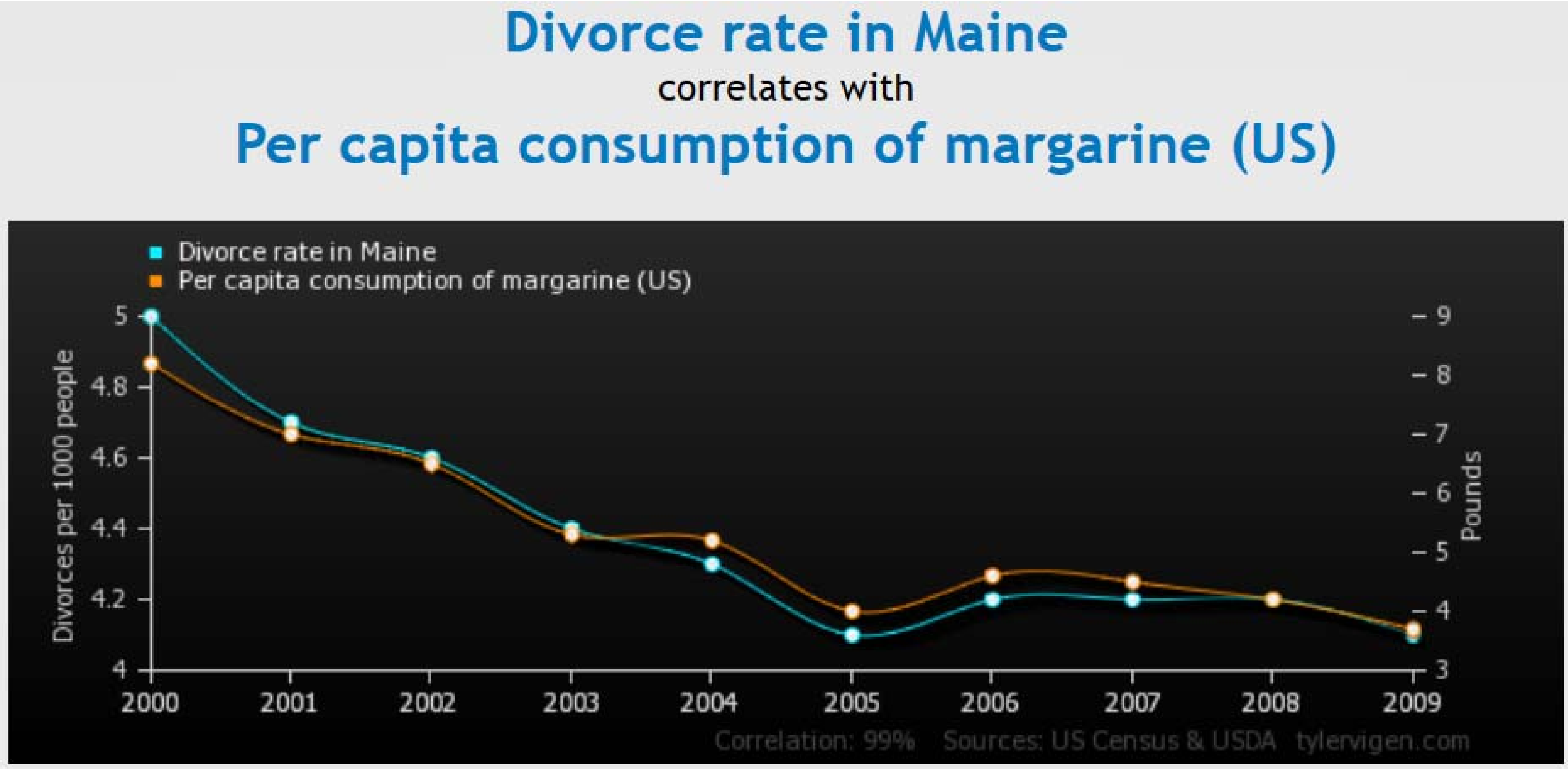
Spurious Correlation



Correlation: 0.992082

http://www.tylervigen.com/

Spurious Correlation



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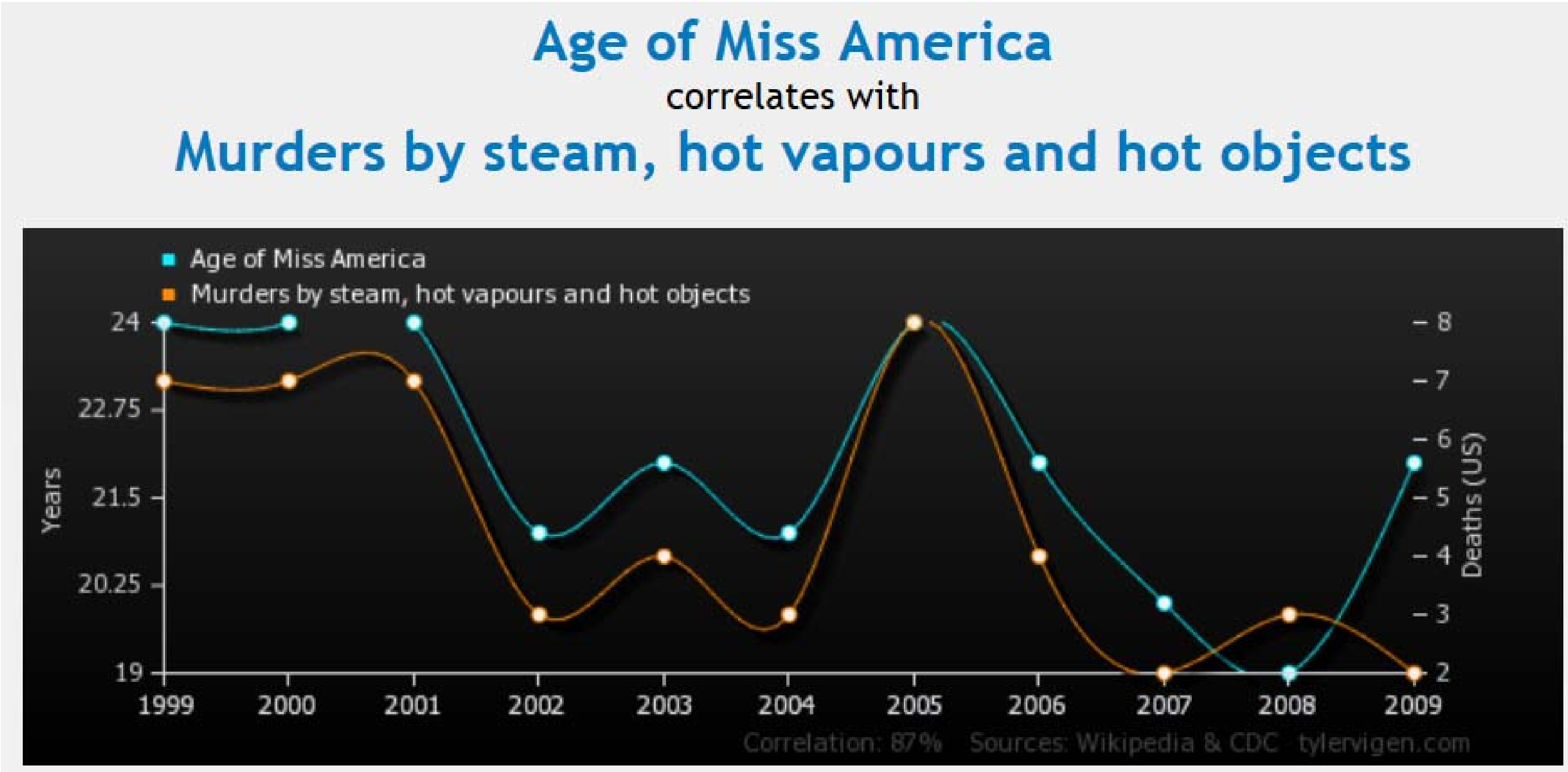
Spurious Correlation



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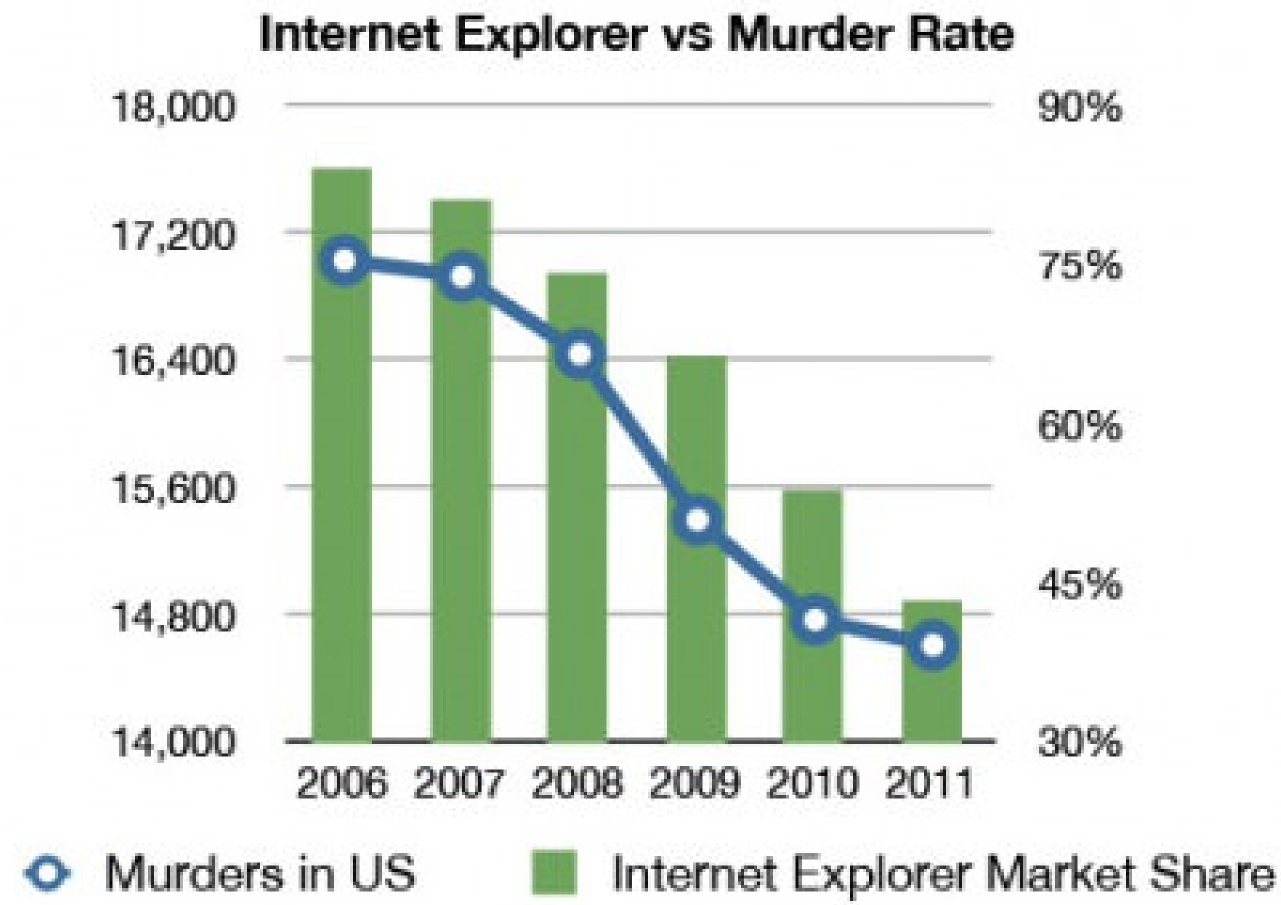
Spurious Correlation



(gross number of murders)

[http://www.geek.com/microsoft/does-internet-explorers-falling-market-share-mirror-the-drop-in-us-homicides-1537095/]

Spurious Correlation